Modeling Choices with Probabilistic Preference and/or Indifference: A Binary Choice Model Based on Two Thresholds*

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This paper develops a binary choice model in light of the notion of probabilistic preference and/or indifference in the preference structure within an information processing framework. Two thresholds based on the difference in utility from two alternatives are theorized to help articulate ordinary, day-to-day choice situations more realistically. The three states including "indifference" replace "preferences" between the stages of simple perceptions and choice. Further, five postulates are formulated based upon the two thresholds, hence the corresponding conditional probabilities derived. These two thresholds with the five postulates are incorporated into the conventional probabilistic choice model. Accordingly, the marginal distribution for the three states are derived. This theoretical model of two thresholds contributes to theorize probabilistic choice as the outcome of probabilistic preference and indifference. The model is applied to a simple capital investment problem based on Net Present Value for the estimation of two parameters, i.e., the two thresholds

Key Words: preference, indifference, threshold, logit model, binary decision

Ⅰ. Introduction

Quite often in reality, choices are made without preference due to decision maker's inability or unwillingness to evaluate all the available alternatives (Ariely 2010); i.e., they simply pick (Rescher 1960). O'Neill (2010) discusses indifference is given "a praxeology interpretation." It may be due to the fact that alternatives may appear similar each other and may not offer any distinctive differences. Consequently, decision makers may make a random choice out of the state of indifference, i.e., without preference if in a mandatory choice situation; even if a mandatory one, they may do so due to the fear of procrastination or the need for simply completing the decision itself. This paper attempts to incorporate these phenomena into a probabilistic choice model in mandatory choice situations, for simplicity, binary choice situations.

The investigation of how individuals judge and choose among different alternatives has

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long attracted a great number of consumer researchers in a variety of disciplines such as marketing, psychology, economics, and decision sciences (e.g., Bettman 1986; Chintagunta and Nair 2011; Corstjens and Gautsch 1983; Einhorn and Hogarth1981, Kardes, Herr and Haugtvedt 2008; Meyer and Kahn 1989 Rossi, Allenby and McCulloch 2005; Takemura 2014; Train 2003; Zheng and Jin 2016). Study on evaluative judgment and choice is expected to continue sparkling considerable interests among researchers consistently as a fundamental subject matter in marketing, psychology and decision-making disciplines. This article attempts to investigate more closely and intuitively the decision processes of consumer judgment and choice in light of the notion of indifference and preference in the preference structure within an Information Processing paradigm (IP paradigm).

Recent years have witnessed some progress in consumer judgment and choice research, the deviations from traditional approaches in terms of the unit of decision maker for a single choice situation and the comparability of choice alternatives, i.e., family or group decision making rather than individual decision making (Curry and Menasco 1979; Koo 2006; Saaty and Peniwati 2013) and choice of non-comparable, rather than immediately and directly comparable, alternatives (Kim Cho, Khan and Ravi 2013; M. Johnson 1988, 1984). However, choice and decision making of an individual facing with comparable alternatives seem to remain still puzzled and unexplored to a great extent. The main purpose of this article is to present a theoretical model of individual judgment and choice between directly and immediately comparable alternatives where choice is explained as the outcome of indifference and/or preference which constitute the preference structure, apart from the choice structure, in overall choice process within an information processing paradigm. The two threshold levels expressed as the utility difference between the two alternatives are introduced and incorporated into the conventional logit model.

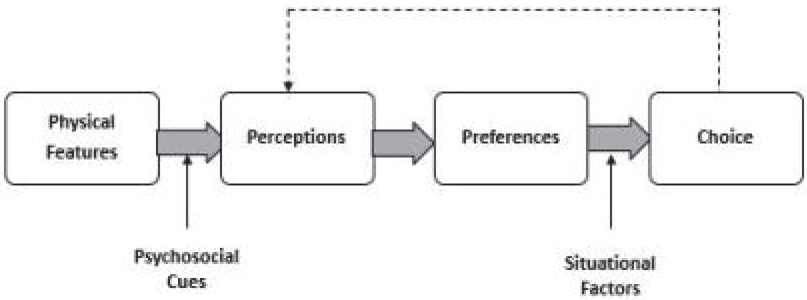
Ⅱ. Literature Review

2.1 Information Processing Model of Consumer Decision Making

In marketing information processing (IP) may be the most dominant paradigm about how consumers receive, process, and retain information to make decision or to reach choice outcome. In specific, the "lens" model developed by Brunswick (1952) has provided the theoretical basis for describing consumer decision processes in previous studies (e.g., Moore and Winer 1987; Hauser and Simmie 1981; Zeng and Jin 2016); a similar version of the simplified model is illustrated in 〈Figure 1〉 where the four distinctive stages in choice process are identified. In this model physical features of the alternative are perceived by individual consumer; but these perceptions are almost always moderated and mediated through the psychosocial cues such as advertising and peer influences. Then these filtered perceptions retained by consumer form a basis for preferences, which eventually affect consumer choice subject to certain observable constraints such as time and budget, product availability, and promotion. With buying and consuming experiences of the product consumer in turn may update the perceptions and adjust the preferences and choice behavior.

2.2 Preference and Choice Structures in Information Processing Paradigm

Hauser and Simmie (1981) attempt to model the linkage from physical features to perceptual dimensions which precede preferences; thus, a mapping from physical characteristics to a perceptual space is investigated. The intervening stage of perceptions is incorporated into the Lancasterian economic model of consumer behavior (Ladd and Zober 1977; Lancaster 1966, 1971; Ratchford 1979; Peitz 1997) with psychometric techniques of the measurement of the functional mapping of the physical features into the perceptual dimension space. Consequently, mapping from product characteristics to perceptions and perceptions to preferences are explicitly treated. However, it is noticed that although preferences and choice are treated as the two separate stages in Information Processing paradigm as in ⟨Figure 1⟩, the relationship between preferences and choice is not explicitly addressed in except for the constraints intervening between the two (e.g., Moore and Winer 1987; and Hauser and Simmie 1981). Preferences and choice are in general treated as equal yet differ only with the interference of situational factors ((Muthukrishnan and Kadres (2001). In other words, the preference structure is treated equivalent to a choice structure under the nonexistence of situational constraints investigated (Miller and Ginter; 1979). Yet, Blin and Dodson (1980) discuss about preference vs. choice and present a theory of choice which is linked directly to the consumer's attitude structure. Thus, they claim to provide a basis that attitudinal measures (perceptions and preferences) are related to a choice be-



⟨Figure 1⟩ Simplified Lens Model

havior, which can be represented as an integral and rational element of consumer behavior without having to appeal to "situationalism," and add randomness which contributes to the inconsistency between preference and choice structure.

2.3 Previous Threshold Models

2.3.1 Threshold in Utility-based Stochastic Choice Models

Utility-based stochastic choice models have integrated a single threshold into the conventional logit models. In conventional logit models (Amemiya 1981; Train 1987; Ben-Akiva and Lerman 1985; Chintagunta 2011; Maddala 1983; McFadden 1973, 1982;) distinct utilities can be derived and quantified in accordance with a specified set of criteria which accommodates the decision maker's expressed preferences. He is then assumed to select an alternative which offers the greatest utility no matter how small the difference is.

In practice, if the two respective utilities attributed to the two given alternatives are very similar, the validity of the assumption is questionable. If the two utilities are so similar that their difference is perceived as negligible, the decision maker would be insensitive as to this difference and indifferent to the two alternatives. Thus, a certain threshold has to be exceeded in order for him to perceive the two alternatives as noticeably different and to prefer one alternative to another. Hence, the notion of a threshold has been acknowledged and incorporated into the conventional logit models; hence threshold have been developed and applied mostly in the transportation research areas (Cantillo 2010; Krishnan 1975, 1977; Lioukas 1984; Wang et al. 2016). This threshold serves as a basis for a decision maker to differentiate an alternative from others in situations where competing alternatives offer similar values in utility. It is of surprise that no research has been done in the areas of management where decision makers, whether individual or organization, face many similar alternatives.

Krishnan (1977) formalized threshold as "minimum perceivable difference" between the utilities of the alternatives compared. According to this formalization the decision maker would prefer one alternative to another if the utility of the former exceeds that of the latter at least by the threshold. If, however, the excess of utility is less than the threshold, a state of indifference would result. He incorporated the threshold into the conventional binary logit model. Lioukas (1984) took into consideration the threshold to describe the problem when the decision maker is confronted with more than two alternatives. In an extension of Krishnan's binary logit theory to multiple-alternative situations, he applied the multinominal logit model to study the problem and incorporated the threshold concept into the multinominal logit model.

In both studies by Krishnan (1975, 1977) and Lioukas (1984) a single threshold in discriminating differences of utilities is assumed to provide a basis for delineating the choice behavior of decision maker. Although the study by Lioukas (1984) investigated the situation where there are more than two alternatives and further systematized the treatment of threshold and threshold-associated intransitivities, both studies characterized the existence of a single threshold incorporated into the conventional logit model.

2.3.2 Implications and Problems of Single Threshold Models

The two models by Krishnan (1977) and by Lioukas (1984) assume the existence of threshold, a single threshold. These single threshold models are intrinsically based on the three postulates of preference corresponding to the three levels of preference state inherent within an individual decision maker. Provided that A and B are the two alternatives and U_A and U_B, their respective utilities, the three postulates of preference are given as follows:

(i) A is preferred to B if $U_A - U_B > \delta$
(ii) B is preferred to A if $U_B - U_A > \delta$
(iii) A is equally preferred to B if $|U_A - U_B| < \delta$.

Each of the first two postulates specifies that one alternative is explicitly preferred to another if the utility of the first exceeds the utility of the second at least by the threshold δ, a positive constant. The third postulate specifies that decision maker is indifferent between the two alternatives if the absolute value of the difference in utility values is less than the threshold.

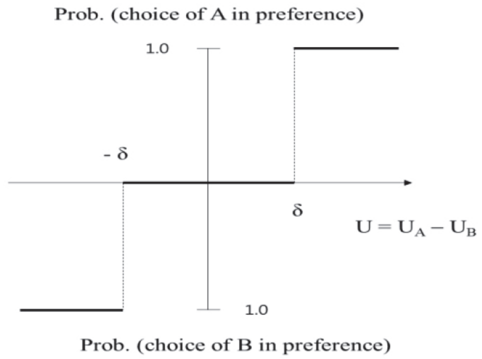
The three postulates of choice behavior were formulated in connection with those of preference. These postulates are given as follows:

(i) If A is preferred to B, then A will be chosen with probability 1.
(ii) If B is preferred to A, then B will be chosen with probability 1.
(iii) If A is equally preferred to B, then A and B will be chosen with probabilities Θ and (1 - Θ), respectively.

Each of the first two postulates specifies that if an alternative is preferred then it will be chosen with certainty. The third postulate specifies that in case of indifference A and B will be chosen randomly. However, this random choice in case of indifference is valid only under the assumption that a choice of either one is mandatory for the decision maker. If this assumption is not stipulated, a state of pure indifference may cause the decision maker not to make any choice. This possibility is noteworthy in that the binary choice situation may, without the above assumption, may turn out to be trinary outcome--the choice of A, the choice of B, and no choice. Henceforth, a mandatory choice explicitly accompanies our treatment of the binary choice situation--the choice of A or the choice of B. In addition, observing the two sets of postulates, one on preference and another on choice behavior, we find the preference and choice to be equivalent concepts -- choice structure is dictated by preference structure.

Also, with reference to the two sets of postulates the following inferences can be made. First, a state of being "preferred" leads to the choice with a probability 1 of one alternative over another. Second, a state of being "equally preferred" in actuality, induces pure random choice without implying any degree of preference under mandatory choice situation; as mentioned earlier, the possibility of no choice is precluded. That A and B will be chosen randomly with probabilities Θ and (1 - Θ) respectively, means that either one can be chosen in absence of a state of being "preferred." Thus, the "choice" can be of more than one form; it does not necessarily accompany a state of being "preferred." The "choice" may also accompany a state of being "indifferent" (Machaj 2007; Margalit-Ullmann and Morgenbesser 1977; O'Neill 2010; Rescher 1960). Finally, as far as the "choice in preference" (the choice resulted from a state of being preferred) and its complement are concerned, choice functions over the possible situations $U_A - U_B > \delta$, $U_B - U_A > \delta$, and $|U_A - U_B| < \delta$ show discreteness. ⟨Figure 2⟩ attempts to delineate the choice functions conditional on possible situations. U refers to the magnitude of difference between U_A and U_B, i.e., $U_A - U_B$. In this graphical delineation the horizontal axis implies the state of being "indifferent" where a purely random choice is made rather than a choice in preference. Given $|U_A - U_B| < \delta$ the probability that A is chosen in preference (the choice resulted from a state of "preferred") is 0, and also the probability that B is chosen in preference (the choice resulted from the state of "preferred") is 0. This is denoted as the following:

$$P(\text{Choice of A in preference} \setminus |U_A - U_B| < \delta) = P(\text{Choice of B in preference} \setminus |U_A - U_B| < \delta) = 0 \quad (1)$$



⟨Figure 2⟩ Choice Function under a Single Threshold

Note that given $|U_A - U_B| < \delta$, A and B will still be chosen without any preference or motive with probabilities Θ and (1 - Θ) merely through some random process since a mandatory choice is assumed.

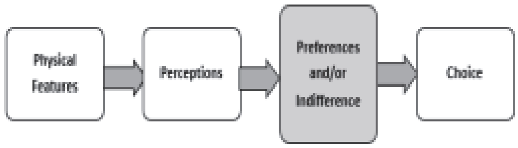
The possible choice states as to the three possible situations are identified "choice of A in preference," "choice of B in preference," and "being indifferent (thus, random choice of either one)." The conditional probabilities characterizing the three situations can be elaborated as the following:

1. Prob. (Choice of A in preference \ $U_A > U_B + \delta$) = 1.0 (2a)
   Prob. (Choice of B in preference \ $U_A > U_B + \delta$) = 0 (2b)
   Prob. (Being indifferent \ $U_A > U_B + \delta$) = 0 (2c)
2. Prob. (Choice of A in preference \ $U_B > U_A + \delta$) = 0 (3a)
   Prob. (Choice of B in preference \ $U_B > U_A + \delta$) = 1.0 (3b)
   Prob. (Being indifferent \ $U_B > U_A + \delta$) = 0 (3c)
3. Prob. (Choice of A in preference\ $|U_A - U_B| < \delta$) = 0 (4a)
   Prob. (Choice of B in preference \ $|U_A - U_B| < \delta$) = 0 (4b)
   Prob. (Being indifferent \ $|U_A - U_B| < \delta$) = 1 (4c)
   thus, Prob. (Random choice of A \ $|U_A - U_B| < \delta$) = Θ (4d)
   Prob. (Random choice of B \ $|U_A - U_B| < \delta$) = $1 - \Theta$. (4e)

Equations (4a) and (4b) are given as the same as Equation (1); it is repeated for clarity and consistence in the context of equations.

Ⅲ. Development of Conceptual Framework

The information processing (IP) paradigm provides the conceptual framework for the model; the feedback effect from post-choice experience to perceptions would not be considered as in Hauser and Simmie (1981) and Moore and Winer (1987). What appears to be strikingly different from the previous IP paradigm as in 〈Figure 1〉 is that preference structure is the combination of preference and indifference. 〈Figure 3〉 illustrates the conceptual framework of the model. Whether consumer faces all alternatives simultaneously or sequentially, it is assumed that all the alternatives are available at the time of choice; and consumer must choose one over the other, i.e., mandatory choice among available alternatives. Also, for the development of the simple model only two alternatives are assumed to exist for choice, i.e., binary choice. In practice, the transportation mode choice exemplifies this condition, e.g., choice of private or public transportation (Ben-Akiva and Lerman 1985, esp. Chapter 5; Krishnan 1975).



〈Figure 3〉 Conceptual Framework of the Model

3.1 Preference and Indifference

The core of the conceptual framework relies on the notion of choice without preference (Hoppe, 2005; Machaj 2007; Rescher 1960; O'Neill 2010; Ullmann-Margalit and Morgenbesser

1977;). The problem of choice with or without preference has been debated mostly as a philosophical inquiry. The choice without preference has been epitomized in the history and logic of the problem of "Buridan's ass" (Rescher 1960). Although the problem appears as philosophical inquiry, Ullmann-Margalit and Morgenbesser (1977) claim that there are genuine "picking" situation -- where preferences are completely symmetrical and thus one is strictly indifferent with regard to the alternatives. O'Neill (2010) discusses that "indifference and choice are surprisingly tricky issues in economics . . . within the literature of the Austrian school" and shows that "the alternative approach of using non-strict preference preferences allows indifference to be given a praxeological interpretation."

Further, we may inquire if there exist only two mutually exclusive and exhaustive states -- (absolute) preference and indifference (no preference) -- prior to choice, i.e., absolute preference which leads to a choice with preference or indifference which leads to a choice without preference, in other words, with indifference. Rather, there must exist some intermediate state between preference and indifference, viz., partial preference or weak preference which does not always lead to a choice of one over another with confidence or with absolute preference since preference and indifference could be hardly recognized as two polar, dichotomous concepts, but rather reasonably as a continuum.

3.2 Preference Structure: Absolute Preference, Partial Preference, and Symmetric Preference

As it is recognized that preference and indifference constitute the preference structure, based on the degree of preference the three levels of preference are identified -- "absolute preference," "partial preference," and "symmetric preference." In this article some terms are used interchangeably: "Absolute preference" would be also called "preference" as opposed to partial preference and indifference; and also, preference is used as generic term referring to any degree of preference; "symmetric preference" is used equivalent to "indifference" or "equal preference"; and partial preference also refers to the coexistence of preference and indifference. In case of absolute preference always one alternative is chosen over another "with motive" (Rescher 1960). Whenever necessary to emphasize the choice resulting from preference and to distinguish the choice in random resulting from indifference or partial preference, "choice with motive" is adopted in the article. In case of symmetric preference either one is randomly chosen or simply "picked" (Machaj 2007; Ullmann-Margalit and Morgenbesser 1977) without motive. Finally, in case of partial preference, i.e., coexistence of preference and indifference in human judgment of preference, it can be either that always one alternative is perceived to be preferred over another and chosen with motive or that the two alternatives are perceived to be indifferent and either one is randomly chosen over the other without motive.

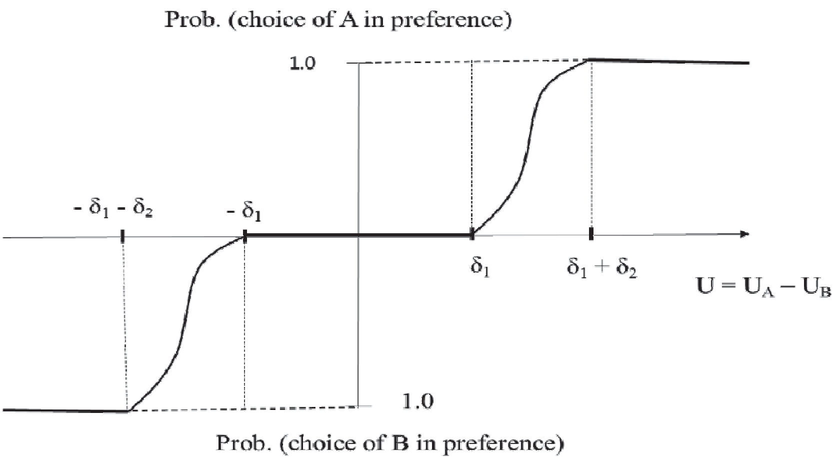
Ⅳ. Double Threshold Model

The main objective of this study is to develop within an IP paradigm, a binary stochastic choice model based on the assumption of two thresholds. This two- threshold assumption would replace the single threshold assumption (Krishnan 1974, 1977; Lioukas 1984) in binary choice situations. It would serve to elaborate the distinction between preference structure and actual choice behavior in IP paradigm of consumer decision making. Explicit probabilities of preference and indifference in preference structure are derived, which constitute the basis of actual choice probabilities. Thus, trinary states in preference structure, i.e., "A preferred," "B preferred," and indifferent are identified albeit the actual choice state is binary, i.e., "A chosen," and "B chosen."

4.1 Assumption of Two Thresholds

The two sets of postulates on which the two models by Krishnan (1977) and by Lioukas (1984) are based, pose some problem with respect to the discreteness of the choice functions. It is distinguishable that the choice of A or B in preference does not occur with some probability less than 1 but does always with a probability 1. 〈Figure 2〉 shows that at the point $-\delta$ and δ on horizontal axis U a transition is made from choice of B in preference to being indifferent and at the point δ another, from being indifferent to choice of A in preference. There exists on horizontal axis U no interval where the choice of B or A in preference with some probability less than 1 occurs. Of course, the probability of random choice, not that of the choice in preference, may have some probability less than 1 in the interval between $-\delta$ and δ on U. Thus, it seems necessary to insert two additional intervals so that the choice in preference of either A or B with some probability less than 1 can be represented. Furthermore, some continuous functions are presumably drawn on the two intervals; this is depicted in 〈Figure 4〉. These intervals are one between δ_1 and $\delta_1+\delta_2$ and another between $-\delta_1-\delta_2$ and $-\delta_1$.

The *raison d'être* of the two thresholds in binary choice situation can be justified through the close investigation of the inherent postulates in a single threshold assumption. As the two alternatives are compared and the magnitude of the difference in two utility values derivable from the two alternatives exceeds the threshold, a small excess over the threshold should be discriminated from a large excess in term of the intensity in preference structure. Although any level of excess over the threshold may cause a state of preference rather than a state of indifference, a large excess would entail a high degree of preference and hence leads to a choice in preference of one alternative over another with a probability of



〈Figure 4〉 Modified Choice Function under Two Thresholds

1. However, a small excess may not yield a high degree of preference and therefore leads a choice in preference obviously with a probability of less than 1; in other words, this weak preference, or the possible coexistence of indifference would yield a choice of one alternative with a probability of less than 1. Thus, in case of a high degree of preference the preference structure is equivalent to the choice structure; however, in case of a low degree of preference the preference structure may not equal to the choice structure due to indifference and random choice from indifference.

The first threshold within which a pure indifference would occur is denoted as δ_1 and the second as δ_2. Thus, when the difference between the two utility values lies between δ_1 and $\delta_1 + \delta_2$, the intensity of preference would increase monotonically. The relation of δ in single threshold model to δ_1 and δ_2 in double threshold model would remain as an empirical question. The small excess over the first threshold can fall between $-\delta_1 - \delta_2$ and $-\delta_1$ or between δ_1 and $\delta_1 + \delta_2$. These intervals are in contrast to the conditional relationship specified in the postulates of choice behavior under the assumption of a single threshold in that an alternative is preferred then it will be chosen with a probability 1. (See equations (2a) to (4e))

The notion of the two thresholds is introduced in binary choice situation; and it is clear that these are positive parameters. In order for an alternative to be preferred to another and chosen (in preference) with a probability 1, the absolute difference between two utility values should exceed $\delta_1 + \delta_2$. When the absolute difference is between δ_1 and $\delta_1 + \delta_2$, an alternative is considered preferred and chosen with probability, say, $\psi(U)$; and $\psi(U)$ has some probability distribution for $\delta_1 < U < \delta_1 + \delta_2$ or $-\delta_1 - \delta_2 < U < -\delta_1$. With a probability $1 - \psi(U)$ this difference results in a state of being indifferent where a random choice instead of a choice in preference is made. Thus, δ_1 would constitute the first threshold. When the absolute difference is less than δ_1, a state of pure indifference would occur; therefore, a choice is made randomly at all times.

The assumption of the two threshold levels enables us to delineate the choice behavior of decision maker more rationally than that of a single threshold. In some binary choice situations, the decision maker may be led to a situation where one alternative is more or less preferred to another but this level of preference does not necessarily imply that the alternative is selected with a probability of 1. This situation corresponds to the case that the absolute difference between two utility values is greater than the first threshold level δ_1 but less than the second threshold level $\delta_1 + \delta_2$. In this case, with a probability (U) (for $0 < (U) < 1$) the decision maker will choose one alternative in preference to another with a probability 1 - (U) he will be indifferent to the two alternatives; then a random choice will be made.

Only when the absolute difference exceeds the second threshold level, one alternative will be preferred and at the same time chosen with a probability of 1 in preference to another. When the absolute difference is less than the first threshold level δ_1, this situation results in a state of indifference where the choice is purely random. Under the assumption of the two threshold levels, a state of indifference may also result from the case that the absolute difference is between the first and the second threshold levels as well as the case that the absolute difference within the first threshold level.

As mentioned previously, a choice is assumed mandatory. No matter what the magnitude of the absolute difference between two utility values is, a choice is at all times made eventually either by a random process or the decision maker's own perception of preference. In case that the absolute difference is less than the first threshold level, with a probability of 1 he is led to a state of indifference where the choice is made purely by a random process; accordingly, the two alternatives will be selected with probabilities Θ and $(1 - \Theta)$, respectively. In case that the absolute difference is between the threshold levels, with a probability 1 - (U) a state of indifference would also occur. This situation yields that the decision maker is also to choose one with a probability Θ and another with $(1 - \Theta)$ by a random process. In addition, with a probability (U) a state of preference would occur, and he will choose one by cognition of preference. In other case that the absolute difference exceeds the second threshold, a state of preference always results; and one alternative is preferred and chosen in preference with a probability of 1.

A possibility of choice from a state of preference without a probability of 1 has not been treated previously. By introducing the second threshold level such a possibility is represented. On the work described here a new model is formulated based on the assumption of the two threshold levels within the framework of binary logit analysis.

For semantic clarity the "threshold levels" are distinguished from the "thresholds" in the DTLM. In terms of notation the former is denoted δ_1 and $\delta_1 + \delta_2$; and the latter, δ_1 and δ_2. In effect, the second threshold is considered a threshold on the first threshold. The two threshold levels function as a basis by which all possible cases in the difference of two utility values compared are distinguished and characterized. Also, to reiterate, a state of preference includes a choice with perfect certainty as well as that with less certainty.

The "five cases" would specifically refer to the five possible situations for which the magnitude of the difference between two utility values may be positioned. The "three conditional states" imply the three distinctive states of preference structure (leading to some actual choice decision) conditional on any of the "five cases." These conditional states are:

A is preferred given case k (6)
B is preferred given case k (7)
A and B are indifferent given case k (8)
where k = i, ii, iii, iv, v and
i = The difference is greater than $\delta_1 + \delta_2$
ii = The difference is less than $-\delta_1 - \delta_2$
iii = The difference is between δ_1 and $\delta_1+\delta_2$
iv = The difference is between $-\delta_1 - \delta_2$ and $-\delta_1$
v = The difference is between $-\delta_1$ and δ_1.

In addition, the "three preference states" are defined as follows:

A is preferred; simply, "A preferred" (9)
B is preferred; simply, "B preferred" (10)
A and B are indifferent; simply, "indifferent." (11)

Accordingly, conditional distributions can be derived for the "three conditional states"; while marginal distributions can be driven for the "three states."

4.2 Two Thresholds and Five Cases

Assuming A and B are two alternatives and U_A and U_B, their respective utilities, the five postulates of preference and choice behavior are made based on the existence of the two threshold levels δ_1 and $\delta_1 + \delta_2$ as the following:

Given $U_A > U_B + \delta_1 + \delta_2$, A is preferred and chosen in preference with a probability 1. (12)

Given $U_B > U_A + \delta_1 + \delta_2$, B is preferred and chosen in preference with a probability 1. (13)

Given $U_B + \delta_1 < U_A < U_B + \delta_1 + \delta_2$, A is preferred and chosen in preference with a probability $\psi_1(U)$ $(0 < \psi_1(U) < 1)$. (14)

Given $U_A + \delta_1 < U_B < U_A + \delta_1 + \delta_2$, B is preferred and chosen in preference with a probability $\psi_2(U)$ $(0\ \psi_2(U) < 1)$. (15)

Given $|U_A - U_B| < \delta_1$, A and B are indifferent and chosen with probabilities Θ and $(1-\Theta)$ (for $0 < \Theta < 1$). (16)

The first two postulates (12) and (13) refer to the situation that the utility of one alternative exceeds that of another by at least the second threshold level $\delta_1 + \delta_2$. In this situation one alternative is preferred to another, and the former is chosen in preference with a probability of 1. The next two postulates (14) and (15) refer to the situation that the utility of one alternative exceeds that of another by at least the first threshold level δ_1 but not by the second threshold level $\delta_1 + \delta_2$. In this situation with a probability ψi (for i = 1, 2) one alternative is preferred to another and chosen in preference. And with a probability $(1 - \psi_i (U))$ a state of indifference occurs; hence the decision maker is indifferent between A and B. However, an actual choice decision is still made by a random process; and A and B will be chosen with probabilities Θ and (1 - Θ). Therefore, the probabilities that A or B is chosen, either in random or in preference, under each case are given as:

$$\text{Prob. (A chosen} \setminus U_B+\delta_1 < U_A < U_B +\delta_1 +\delta_2) = \psi_1(U) + (1 - \psi_1(U))\,\Theta \quad (17)$$

$$\text{Prob. (B chosen} \setminus U_B + \delta_1 < U_A < U_B + \delta_1 + \delta_2) = (1 - \psi_1 (U))\,(1 - \Theta) \quad (18)$$

$$\text{Prob. (A chosen} \setminus U_A + \delta_1 < U_B < U_A + \delta_1 + \delta_2) = (1 - \psi_2(U))\,\Theta \quad (19)$$

$$\text{Prob. (B chosen} \setminus U_A + \delta_2 < U_B < U_A + \delta_1 + \delta_2) = \psi_2(U) + (1 - \psi_2(U))\,(1 - \Theta) \quad (20)$$

The last postulate (16) refers to the situation that the absolute value of the difference in utility is less than the first threshold level δ_1. In this situation A and B are indifferent with a probability 1; thus, the decision maker is indifferent between the two. Then the actual choice refers to the situation that the absolute value of the difference in utility is less than the first threshold level δ_1. In this situation A and B are indifferent with a probability 1; thus, the decision maker is indifferent between the two. Then the actual choice decision is made always by a random process rather than by the outcome of any degree of preference; and A and B will be chosen with probabilities Θ and (1 - Θ), respectively.

From the five postulates the five cases are formally denoted as the following:

$$\text{I) } U_A - U_B \geq \delta_1 + \delta_2 \quad (21)$$

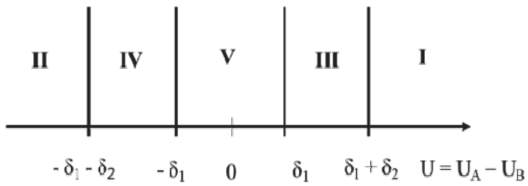
$$\text{II) } U_A - U_B \leq -\delta - \delta_2 \quad (22)$$

$$\text{III) } \delta_1 \leq U_A - U_B < \delta_1 + \delta_2 \quad (23)$$

$$\text{IV) } -\delta_1 - \delta_2 < U_A - U_B \leq -\delta_1 \quad (24)$$

$$\text{V) } -\delta_1 < U_A - U_B < \delta_1 \quad (25)$$

Remind $U_A - U_B$ is denoted by U. Then the five cases are represented by the five regions I, II, III, IV, and V as illustrated in Figure 5.



〈Figure 5〉 Illustration of the Five Regions

4.3 Specification of the Three Conditional States

The three conditional states were previously identified and the five cases i, ii, iii, iv, and v are represented by the five regions I, II, III, IV, and V in Figure 5, respectively. Then the following sets of the conditional probabilities are derived:

1. For the region

$\pi_1(I)$ = Prob. (A preferred \ $U > \delta_1+\delta_2$) $= 1$ (26a)

$\pi_2(I)$ = Prob. (B preferred \ $U > \delta_1+\delta_2$) $= 0$ (26b)

$\pi_{12}(I)$ = Prob. (Indifferent \ $U > \delta_1+\delta_2$) $= 0$ (26c)

2. For region II

$\pi_1(II)$ = Prob. (A preferred \ $U < -\delta_1-\delta_2$) $= 0$ (27a)

$\pi_2(II)$ = Prob. (B preferred \ $U < -\delta_1-\delta_2$) $= 1$ (27b)

$\pi_{12}(II)$ = Prob. (Indifferent \ $U < -\delta_1-\delta_2$) $= 0$ (27c)

3. For the region III

$\pi_1(III)$ = Prob. (A preferred \ $\delta_1 < U < \delta_1 + \delta_2$) $= \psi_1(U)$ (28a)

$\pi_2(III)$ = Prob. (B preferred \ $\delta_1 < U < \delta_1 + \delta_2$) $= 0$ (28b)

$\pi_{12}(III)$ = Prob. (Indifferent \ $\delta_1 < U < \delta_1 + \delta_2$) $= 1 - \psi_1(U)$ (28c)

4. For the region IV

$\pi_1(IV)$ = Prob. (A preferred \ $-\delta_1-\delta_2 < U < -\delta_1$) $= 0$ (29a)

$\pi_2(IV)$ = Prob. (B preferred \ $-\delta_1-\delta_2 < U < -\delta_1$) $= \psi_2(U)$ (29b)

$\pi_{12}(IV)$ = Prob. (Indifferent \ $-\delta_1-\delta_2 < U < -\delta_1$) $= 1 - \psi_2(U)$ (29c)

5. For the region V

$\pi_1(V)$ = Prob. (A preferred \ $-\delta_1 < U < \delta_1$) $= 0$ (30a)

$\pi_2(V)$ = Prob. (B preferred \ $-\delta_1 < U < \delta_1$) $= 0$ (30b)

$\pi_{12}(IV)$ = Prob. (Indifferent \ $-\delta_1 < U < \delta_1$) $= 1$ (30c)

Thus, $\pi_1(j) + \pi_2(j) + \pi_{12}(j) = 1$ (31)

where j = I, II, III, IV, V.

4.4 Derivation of Probabilities Attributed to the Five Regions

For given alternatives A and B their respective utilities U_A and U_A are assumed to be expressible as the sum of a deterministic and a random component:

$$U_i = V_i + \varepsilon_i \qquad (32)$$

where
V_i = deterministic component of the utility U_i
ε_i = random component of the utility U_i
i = A, B.

Within the framework of the logit analysis the random components ε_1 and ε_2 are assumed statistically independent and identically distributed according to the double exponential distribution (Ben-Akiva and Lerman 1984;

Krishnan 1977, Lioukas 1984, Luce 1959, McFadden 1973, Yellot 1977). This distribution has a density function in a most simplistic form:

$$f(\varepsilon) = \exp(-\varepsilon)\exp(-\exp(-\varepsilon)) \quad (33)$$

where $-\infty < \varepsilon < \infty$.

The cumulative distribution function is given by

$$F(\varepsilon) = \exp(-\exp(-\varepsilon)). \quad (34)$$

When the utilities of the two alternatives are expressed as the sum of their respective deterministic and random components, for each of the five regions in Figure E the following expressions are easily identified:

1. For the region I
$$U = U_A - U_B = V_A + \varepsilon_A - V_B - \varepsilon_B \geq \delta_1 + \delta_2 \quad (35)$$
2. For the region II
$$U = U_A - U_B = V_A + \varepsilon_A - V_B - \varepsilon_B \leq -\delta_1 - \delta_2 \quad (36)$$
3. For the region III
$$\delta_1 \leq U = U_A - U_B = V_A + \varepsilon_A - V_B - \varepsilon_B < \delta_1 + \delta_2 \quad (37)$$
4. For the region IV
$$-\delta_1 - \delta_2 < U = U_A - U_B = V_A + \varepsilon_A - V_B - \varepsilon_B \leq -\delta_1 \quad (38)$$
5. For the region V
$$-\delta_1 < U = U_A - U_B = V_A + \varepsilon_A - V_B - \varepsilon_B < \delta_1 \quad (39)$$

Under the known distribution of the random components ε_A and ε_B, probabilities attributed to the five regions are derivable:

1. For the region I
$$P(I) = 1/\{\exp(-(V_A - V_B) + \delta_1 + \delta_2) + 1\} \quad (40)$$
2. For the region II
$$P(II) = 1/\{\exp((V_A - V_B) + \delta_1 + \delta_2) + 1\} \quad (41)$$
3. For the region III
$$P(III) = 1/\{\exp((V_A - V_B) + \delta_1) + 1\} - 1/\{\exp(-(V_A - V_B) + \delta_1 + \delta_2) + 1\} \quad (42)$$
4. For the region IV
$$P(IV) = 1/\{\exp((V_A - V_B) + \delta_1) + 1\} - 1\{\exp((V_A - V_B) + \delta_1 + \delta_2) + 1\} \quad (43)$$
5. For the region V
$$P(V) = 1 - 1/\{\exp(-(V_A - V_B) + \delta_1) + 1 - 1/\{\exp((V_A - V_B) + \delta_1) + 1\} \quad (44)$$

where $P(j)$ (j=I, II, III, IV, V) refer to the probabilities that the cases represented

4.5 Derivation of Marginal Distribution for the Three States

Marginal distribution is derived for the three states, i.e., "A preferred," "B preferred" and "indifferent," simply by the theorem of total probability (Ross 2013):

1. For "A preferred"
$$\pi_1^* = \sum_{j=1}^{V} \text{Prob. (A preferred)}$$

$$= \sum_{j=1}^{V} \text{Prob.(A preferred\textbackslash Case j)}$$
$$\text{Prob.(Case j)}$$
$$= \pi_1(I)P(I) + \pi_1(II)P(II) + \pi_1(III)P(III) + \pi_1(IV)P(IV) + \pi_1(V)P(V)$$
$$= 1 \bullet P(I) + 0 \bullet P(II) + \psi_1 P(III) + 0 \bullet P(IV) + 0 \bullet P(V)$$
$$= P(I) + \psi_1 P(III)$$
$$= 1/\{\exp(-(V_A - V_B) + \delta_1) + 1\} + (1-\psi_1)/\{\exp(-(V_A - V_B) + \delta_1 + \delta_2) + 1\} \quad (45)$$

Likewise, Prob. (B preferred) can be derived as follows:

2. For "B preferred"

$$\pi_2^* = P(II) + \psi_2 P(IV)$$
$$= 1/\{\exp((V_A - V_B) + \delta_1 + \delta_2) + 1\} + (1-\psi_2)/\{\exp((V_A - V_B) + \delta_1) + 1\} \quad (46)$$

3. For "indifferent"

$$\pi_{12}^* = 1 - \pi_1^* - \pi_2^*$$
$$= (1-\psi_1)/\{\exp(-(V_A - V_B) + \delta_1) + 1\} + (1-\psi_1)/\{\exp(-(V_A - V_B) + \delta_1 + \delta_2) + 1\} - \psi_2/\{\exp((V_A - V_B) + \delta_1 + \delta_2) + 1\} - (1-\psi_2)/\{\exp((V_A - V_B) + \delta_1) + 1\} \quad (47)$$

Finally, the probabilities that A and B will be actually chosen are derived:

1. Prob. (A chosen) $= \pi_1^* + \Theta\, \pi_{12}^*$

$$= (1-\Theta)\psi_1/\{\exp(-(V_A - V_B) + \delta_1) + 1\} + (1-\Theta)(1-\psi_1)/\{\exp(-(V_A - V_B) + \delta_1 + \delta_2) + 1\} - \Theta\psi_2/\{\exp((V_A - V_B) + \delta_1 + \delta_2) + 1\} - \Theta(1-\psi_2)/\{\exp((V_A - V_B) + \delta_1) + 1\} + \Theta \quad (48)$$

2. Prob. (B chosen) $= \pi_2^* + (1-\Theta)\, \pi_{12}^*$

$$= -(1-\Theta)\psi_1/\{\exp(-(V_A - V_B) + \delta_1) + 1\} - (1-\Theta)(1-\psi_1)/\{\exp(-(V_A - V_B) + \delta_1 + \delta_2) + 1\} - \Theta\psi_2/\{\exp((V_A - V_B) + \delta_1 + \delta_2) + 1\} + \Theta(1-\psi_2)/\{\exp((V_A - V_B) + \delta_1) + 1\} + (1-\Theta) \quad (49)$$

Ⅴ. Application

The model is applied to a general capital budgeting situation where the decision maker confronts two alternative investment projects; and he must choose between the two. In order for him to choose one alternative in preference to another with probability 1, the utility of the former should exceed that of the latter by at least a positive constant $\delta_1 + \delta_2$. This situation can occur only when the difference between the utility values falls within the region I or II in ⟨Figure 5⟩. The application is simple practice mainly to find out the values of δ_1, δ_2 rather than to explore the whole picture about how model works.

In capital investment analysis Net Present Value (NPV) has been generally accepted as the basic analytical tools along with Internal Rate of Return (IRR) albeit not with some reservations (Chen and Moore 1982; Eschenbach et al. 2015; Giacotto 1984; Kim and Sonu 2011; Levy and Sarnat 1995). In case of two competing alternatives of capital investment, once NPVs for the two have been determined, the decision criterion is to accept an alternative which brings greater NPV. This conventional criterion can be elaborated by the application of the developed model.

The utilities that the decision makers derive from two alternatives are assumed to be some function g of their respective NPVs. Then the deterministic component of the utility is:

$$V_i = g\ (NPV_i)\ \text{for } i = 1, 2 \qquad (50)$$

Where V_i = the estimated deterministic utility of project i

NPV_i = the observed Net Present Value of project i

The difference between the deterministic components of the utilities is:

$$V_1 - V_2 = g\ (NPV_1) - g\ (NPV_2) \qquad (51)$$

Suppose that the decision maker confronts two alternatives of mutually exclusive capital investments A_1 and A_2. In applying the model, it is assumed that the deterministic component of the utility value V is solely dependent upon the Net Present Value NPV. Then we require the utility function g (NPV) in the equation (51).

Previously, an approach to explain the decision maker's behavior in case of risky investment by way of utility theory was attempted (Giacotto 1984); and samples of the actual utility functions of decision makers were reported relative to real monetary terms. Most of the actual functions from the research were shown nonlinear in nature. Meanwhile, some others were shown almost linear and may be reasonably fitted into a linear function (Giacotto 1984). Through Swalm's data (Swalm 1966), a typical utility function was determined to have the coefficients of 0.22 and the intercept of -0.16. Therefore, the utility function g (NPV) might be expressed as

$$V_i = -0.16 + 0.22\ NPV_i\ \ \text{for } i = 1, 2 \qquad (52)$$

Since the function g is linear, the difference between the two Vs is also a linear function of the difference between the two NPVs. Therefore, it is easily shown that

$$V_1 - V_2 = 0.22\ (NPV_1 - NPV_2) \qquad (53)$$

In actuality, Net Present Value is perceived as a random variable rather than a constant (Hillier 1969); and normal distribution may be assumed with known mean and variance such as:

$$E\ (NPV_1) = \mu_1$$
$$E\ (NPV_2) = \mu_2$$
$$Var\ (NPV_1) = \sigma_1^2$$
$$Var\ (NPV_2) = \sigma_2^2 \qquad (54)$$

Then $NPV_1 - NPV_2$ is also a random variable normally distributed. Under the implicit assumption of the stochastic independence between NPV_1 and NPV_2, the mean μ and the variance σ^2 for $NPV_1 - NPV_2$ are estimable:

$$\mu = \mu_1 - \mu_2$$
$$\sigma^2 = \sigma_1^2 - \sigma_2^2 \quad (55)$$

According to normal distribution the observed values $NPV_1 - NPV_2$ vary as the following:

$$NPV_1 - NPV_2 = \mu + z_k \sigma \quad \text{for } k = 1, 2, 3, \ldots$$

where z_k = a standardized normal random number (56)

The z_k, standardized normal random number, will be generated by a computer routine. Also $V_1 - V_2$ the difference between the two estimated deterministic parts of utility is given as:

$$V_1 - V_2 = 0.22\,(\mu + z_k \sigma) \quad (57)$$

We should note that the data generated here (the values for $V_1 - V_2$ which denote the difference between the deterministic parts of utilities perceived among many individual decision makers) are considered aggregate in nature and as such they may provide rough approximation to disaggregate individual-based choice model (McFadden 1973; Domencich 1975).

The application illustrates how large the magnitude of the difference in term of utility value will be for the decision maker to choose one alternative in preference to another with certainty given μ and σ. Thus, the value of $\delta_1 + \delta_2$ should be found. Substitution of $V_1 - V_2$ by $0.22\,(\mu + z_k \sigma)$ from (57) in equations (45) through (47) leads to the following probabilities for the three states:

1) For "A_1 preferred"

$$\pi_{1k}^* = 1 / \{\exp(-0.22(\mu + z_k \sigma) + \delta_1) + 1\} + (1 - \psi_1)/\{\exp(-0.22(\mu + z_k \sigma) + \delta_1 + \delta_2) + 1\} \quad (58)$$

2) For "A_2 preferred"

$$\pi_{2k}^* = 1 / \{\exp(0.22(\mu + z_k \sigma) + \delta_1) + 1\} + (1 - \psi_2)/\{\exp(0.22(\mu + z_k \sigma) + \delta_1 + \delta_2) + 1\} \quad (59)$$

3) For "indifferent"

$$\pi_{12k}^* = 1 - \pi_{1k}^* - \pi_{2k}^* \quad (60)$$

Thus, we may construct the likelihood function L of the 100 samples in equation (56) based on equations (58) through (60):

$$L = \prod_{k=1}^{100} \pi_{1k}^* \, \pi_{2k}^* \, \pi_{12k}^* \quad (61)$$

Then we ask what values of parameters δ_1, δ_2, ψ_1, ψ_2 would maximize the probability L of obtaining the normality of the data z_k σ. Certainly, these maximizing values of the parameters would seemingly be a good estimate of the parameters because they provide the largest probability of this normality. Thus, we maximize L. ψ_1, ψ_2 should lie between 0 and 1 since they are probabilities. δ_1, δ_2 are two thresholds and must be some positive constant.

For the purpose of comparison with previous single threshold models especially exemplified by Lioukas (1984) we reconstruct some new equations which correspond to (58), (59) and (60) as the following:

1) For "A_1 preferred"

$$\pi_{1k}{}^{*} = 1 / \{\exp(-0.22(\mu + z_k \sigma) + \delta) + 1\} \tag{62}$$

2) For "A_2 preferred"

$$\pi_{2k}{}^{*} = 1 / \{\exp(0.22(\mu + z_k \sigma) + \delta_1) + 1\} \tag{63}$$

3) For "indifferent"

$$\pi_{12k}{}^{*} = 1 - \pi_{1k}{}^{*} - \pi_{2k}{}^{*} \tag{64}$$

The nonlinear likelihood function is established as an objective function to be maximized subject to the constraints for δ_1, δ_2, ψ_1, ψ_2. A successive linear programming (SLP) technique was employed. Table 1 illustrates how the value for $\delta_1 + \delta_2$ varies as μ varies given $\sigma = 30$ and $\psi_1 = \psi_2 = 0.5$. Likewise, it also provides how the value for δ varies under the single threshold model. Note that ψ_1 and ψ_2 in the current model with two thresholds do not exist in the model with a single threshold. The results indicate that, first of all, δ is greater than δ_1 and far less than $\delta_1 + \delta_2$, which is not inconsistent with the intuition; and it is closer to δ_1 rather than $\delta_1 + \delta_2$. This implies that the pure indifference region, the case V in the equation (25) and illustrated in the Figure 5.

Second, -log L (max) in the current model is slightly smaller than that in the single threshold model which clearly shows the better fitting with the double threshold model. However, it may be argued that the double threshold model obviously sacrifices parsimony compared to the single model since the former has one more parameter than the latter. If we consider the improvement in the model fitting for the single threshold model was more significant as compared the traditional logit model without a threshold (refer to the pp. 117~118 in Lioukas 1984). Thus, it must be recapitulated that the main contribution of the new model lies not in the improvement of the single threshold with the addition of another threshold level for a marginal improvement through a better model fitting. Focus should be given to the fact that

〈Table 1〉 Estimates of Parameters

INPUT		OUTPUT					
		Double Threshold Model				Single Threshold Model	
σ	μ	δ_1	δ_2	$\delta_1 + \delta_2$	-log L (max)	δ	-log L (max)
30	0	5.833	5.833	11.666	726.65	7.648	755.72
30	5	5.966	5.966	11.932	717.34	7.994	746.16
30	10	6.249	6.249	12.498	713.93	8.624	739.69
30	15	8.159	8.159	16.318	703.85	11.586	731.05

the double threshold model presents a novel approach of a theoretical model of individual judgment and choice between directly and immediately comparable alternatives where choice is explained as the outcome of indifference and/or preference which constitute the preference structure, apart from the choice structure, in overall choice process within an information processing paradigm as presented in Figure 3. Especially, the still debated issue of the notion of indifference is clearly imbedded in the preference structure in that preferences and choice are treated as the two separate stages in the Information Processing paradigm.

Ⅵ. Conclusion and Further Research

The main objective of this study is to develop within an IP paradigm, a binary probabilistic choice model based on the assumption of two thresholds. It provides a new, intuitive perspective on the decision process of consumer judgments and choice in light of the notion of indifference and preference in the preference structure. Two thresholds are proposed to delineate choice situations where different alternatives offer minimally discernible differences. The developed model incorporates these two thresholds within the conventional stochastic choice model. The model also complements the inherent deficiency of the prior "single threshold" logit models which are intrinsically based on the two sets of postulates, one on preference and another on choice behavior. The model was applied to a general capital budgeting situation where the decision maker confronts two alternative investment projects; and he must choose between the two.

Main contribution of this paper is to introduce the notion of indifference into the preference structure apart from the choice structure, which postulates the introduction of two thresholds. The two thresholds intuitively lead to introduce the two intervals, one between δ_1 and $\delta_1 + \delta_2$ and another between $-\delta_1 - \delta_2$ and $-\delta_1$ graphically illustrated in ⟨Figure 4⟩. Hence, the conditional probabilities derived through equations (26a) to (31), then marginal distributions (40) to (44), and finally the probability distribution for two alternatives (48) and (49).

The developed model is expected to capture the intuitive notion of indifference into preference structure and to explain some possible scenarios in decision making where the difference between two alternatives is perceived minimal or hardly discernable. However, it has many limitations. First of all, for the sake of estimation of mainly two thresholds this model is applied to a hypothetical capital investment situation. And the two thresholds are turn out to be the same. There is no way to verify this result. It may be verified through experimental study with survey data, which is expected to require extensive research design. Also, this model needs to be applied to a real data as in transportation research where a limited number of alternatives as transportation modes are available to a large number of users who compare them by time, money and efforts. It could possibly be applied to the Korean mobile telecommunication industry where 3 major companies compete for market share through marketing variables such as pricing and promotion policies.

Second, this model is applicable only to binary choice situation. Natural sequence is to develop a multinomial model to handle a decision-making problem where more than 2 alternatives are available but minimally differentiable each other.

Third, this model could be applied and compared to "preference reversal phenomena" (Alos-Ferrer et al. 2016; Chu and Chu 1990; Grether and Plott 1979; Hutchinson, Kamakura and Lynch, Jr. 2000; Oliver 2013; Lichtenstein and Slovic 1973, 1971; Loomes and Sugden 1988; Schkade and Johnson 1989). For binary choice situation preference reversal is expected to occur in 〈Figure 5〉 the most in the region V, next in the regions III and IV, and the least or none in the regions I and II.

More than a half century ago, Rescher (1960) describes Buridan's ass, the mythical creature, a hypothetical animal, hungry and positioned midway between essentially identical bundles of hay. It is assumed that there exists no reason why the animal should have a preference for one of the bundles of hay over the other; however, it must eat one or the other of them or else starve. Therefore, it must choose one of the bundles. However, there is, by hypothesis, simply no reason for preferring either bundle. Accordingly, it appears to follow that reasoned choice must be possible in the absence of preference. He discusses thoroughly the history of the problem of Buridan's ass from the early Greek philosophy of Anaximander and Socrates to the philosophy of Leibnitz. It is further noted that the problem serves to illustrate the difference between reasons and motives. When a random selection among different objects is made in case of indifference, there is a reason for some particular selection, viz., the fact that it was vindicated by a random selector, e.g., coin tossing. But there is no preference or psychological motivation of other sorts to incline the decision maker to choose this item instead of its alternatives. Thus, we have reasons for a choice even when there is no motive. However, the debate still goes on.

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확률적 선호와 무차별에 의한 선택모형 구축: 두 한계점을 기초로 한 이원선택모형

최인혁*

요 약

이 논문은 정보처리체계에 내재하는 선호구조에 확률적 선호와 무차별(indifference)의 개념을 제시하고 조망하여 새로운 이원선택모형을 개발한다. 일상적인 선택 상황을 더욱 현실적, 논리적으로 묘사하는 데 도움을 주기 위한 이론으로 두 대안의 효용 차이에 근거한 두 한계점(threshold)이 제시된다. 인지 단계에서 선호단계를 거쳐 선택으로 귀결되는 과정에서 무차별을 포함한 3개 상태가 선호단계의 단순한 선호를 대체한다. 또한, 2개의 한계점에 근거하여 5개의 공준(postulates)이 형성되며 이에 상응하는 조건부 확률이 도출된다. 5개의 공준과 2 개의 한계점은 일반적 확률선택 모형에 통합된다. 따라서 3개 상태에 대한 주변분포가 도출된다. 이 이론적 모형은 확률적 선택이 확률적 선호와 무차별의 결과물이라는 이론을 체계화 하는 데 기여한다. 이 모형은 두 계수, 즉, 두 한계점 추정을 위해 순현재가치에 근거한 간단한 자본투자 문제에 적용된다.

주제어: 선호, 무차별, 한계점, 로짓모형, 이원선택모형

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